Existence of ground state solutions for superlinear and subcritical problems by the method of Nehari manifold

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Abstract. In this work, we present a study of the Method of Nehari Manifold treated in [1] which proposes a well-constructed theory to guarantee the existence of ground state solutions for some elliptic differential equations in Hilbert and Banach spaces. Here, we will apply the abstract results to the following equations:

$$\left\{ \begin{array}{rll} -\Delta u - \lambda u &= f(x,u), & \text{in } \Omega \\ u &= 0, & \text{in } \partial\Omega \end{array} \right. \quad \text{and} \quad \left\{ \begin{array}{rll} -\Delta_p u - \lambda |u|^{p-2}u &= f(x,u), & \text{in } \Omega \\ u &= 0, & \text{in } \partial\Omega \end{array} \right.$$

where $\Omega \subset \mathbb{R}^N$ it's a bounded domain, Δ and Δ_p are the Laplacian e p-Laplacian operators, respectively.

Moreover, in the first equation we have $u \in H_0^1(\Omega)$, that is a Hilbert space, $\lambda < \lambda_1$ where λ_1 denotes the firts Dirichlet eigenvalue of $-\Delta$ in Ω, and $f \in C(\Omega \times \mathbb{R}, \mathbb{R})$ satisfies the growth restriction $|f(x,u)| \leq a(1+|u|^{q-1})$ for some $a > 0$ and $2 < q < 2^*$, recall that $2^* := \dfrac{2N}{N-2}$ if $N \geq 3$ and $2^* := \infty$ otherwise.

In the second equation, we have $u \in W_0^{1,p}(\Omega)$, that is a Banach space, $\lambda < \lambda_1$ which λ_1 denotes the firts Dirichlet eigenvalue of $-\Delta_p$ in Ω, and $f \in C(\Omega \times \mathbb{R}, \mathbb{R})$ satisfies the growth restriction $|f(x,u)| \leq a(1+|u|^{q-1})$ for some $a > 0$ e $p < q < p^*$, recall that $p^* := \dfrac{pN}{N-p}$ if $N \geq 3$ and $p^* := \infty$ otherwise.

Adding some assumptions on f we guarantee the existence of infinite pairs of solutions.

References

[1] SZULKIN, A.; WETH, T. *The method of Nehari manifold.* in: D.Y. Gao, D. Motreanu (Eds.), Handbook of Nonconvex Analysis and Applications, International Press, Boston, 2010.